



Refined translation and scale Legendre moment invariants

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ABSTRACT

Orthogonal Legendre moments are used in several pattern recognition and image processing applications. Translation and scale Legendre moment invariants were expressed as a combination of the approximate original Legendre moments. The shifted and scaled Legendre polynomials were expressed in terms of the original Legendre polynomials according to complicated and time-consuming algebraic relations. In this paper, refined translation-scale Legendre moment invariants are obtained through the exact computation of original Legendre moments which completely remove approximation. Fast straightforward computation of central Legendre moments significantly reduces the computational time. According to the tremendous reduction of the computational complexity, the refined set of Legendre invariants is suitable for large size images. The performance of descriptors is evaluated by using a set of standard images.

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1. Introduction

Legendre moments are one of the orthogonal moments that were introduced by Teague (1980). Since this date, Legendre moments have been used in face recognition (Haddadnia et al., 2001), line fitting (Qjidaa and Radouane, 1999), signal noise removal (Kwan et al., 2004) and ECG signal compression (Kwan and Paramesran, 2004). Like all other orthogonal moments, Legendre moments can be used to represent an image with near zero amount of information redundancy (Teh and Chin, 1988). It is well known that the difficulty in the use of Legendre moments in many applications is due to the required high computational complexity, especially when a higher order of moments is used. To overcome this problem, many efficient algorithms (Mukundan and Ramakrishnan, 1995; Shu et al., 2000; Zhou et al., 2002; Yap and Paramesran, 2005; Yang et al., 2006; Hosny, 2007a,b) have been proposed to effectively reduce the computational complexity.

The invariance of Legendre moments can be achieved through different methods. One of them is the direct method. Direct method is based on using Legendre polynomials. Chong et al. (2004) derived translation and scale Legendre moment invariants through complicated and time-consuming system of algebraic relations. Their method is relying on approximate computation of Legendre moments. The main drawback of this method is the high computational demands especially with big size images and the higher or-

der moments. So, this method is suitable only for a set of binary images of a very small size.

This paper proposes a refined translation-scale Legendre moment invariants by using the direct method. Numerical errors are completely removed by using exact original Legendre moments. Fast straightforward computation of central Legendre moments significantly reduces the computational complexity. Aspect ratio invariants are accurately computed by using the exact central moments. A set of gray level images with different sizes are used in the numerical experiments. The obtained results clearly show the efficiency of the proposed method.

The rest of the paper is organized as follows: Section 2 presents an overview of Legendre moments. Through the next two subsections, there is a detailed description of both translation and scale invariants. The proposed method for refined invariants is described in Section 3. Section 4 is devoted to numerical experiments and the analysis of the computational complexity. The conclusion is presented in Section 5.

2. Legendre moment invariants

The two-dimensional Legendre moments of order $(p + q)$ for image intensity function $f(x, y)$ are defined as (Hosny, 2007a):

$$L_{pq} = \frac{(2p+1)(2q+1)}{4} \int_{-1}^1 \int_{-1}^1 P_p(x)P_q(y)f(x, y) dx dy, \quad (1)$$

The p th-order Legendre polynomial $P_p(x)$ is defined as (Chong et al., 2004):

$$P_p(x) = \sum_{\substack{k=0 \\ p-k=\text{even}}}^p B_{k,p} x^k, \quad (2)$$

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where $x \in [-1, 1]$, and the coefficient matrix $B_{k,p}$ defined as:

$$B_{k,p} = (-1)^{\binom{p-k}{2}} \frac{1}{2^p} \frac{(p+k)!}{\left(\frac{p-k}{2}\right)! \left(\frac{p+k}{2}\right)! k!} \quad (3)$$

Legendre polynomial $P_p(x)$ obeys the following recursive relation (Spiegel, 1968):

$$P_{p+1}(x) = \frac{(2p+1)}{(p+1)} x P_p(x) - \frac{p}{(p+1)} P_{p-1}(x), \quad (4)$$

with $P_0(x) = 1$, $P_1(x) = x$ and $p > 1$. The set of Legendre polynomials $P_p(x)$ forms a complete orthogonal basis set on the interval $[-1, 1]$. A digital image of $M \times N$ pixels with intensity function $f(x_i, y_j)$ is mapped into the square $[-1, 1] \times [-1, 1]$ where, $1 \leq i \leq M$ and $1 \leq j \leq N$. For this discrete-space version of the image, Eq. (1) is usually approximated by using zeroth-order approximation (ZOA) as follows (Liao and Pawlak, 1996):

$$\tilde{L}_{pq} = \lambda_{pq} \sum_{i=1}^M \sum_{j=1}^N P_p(x_i) P_q(y_j) f(x_i, y_j), \quad (5)$$

where

$$\lambda_{pq} = \frac{(2p+1)(2q+1)}{MN}. \quad (6)$$

2.1. Translation invariants of Legendre moments

The $(p+q)$ th-order central 2D Legendre moments are defined as:

$$\varphi_{pq} = \frac{(2p+1)(2q+1)}{4} \int_{-1}^1 \int_{-1}^1 P_p(x-x_0) P_q(y-y_0) f(x, y) dx dy \quad (7)$$

where (x_0, y_0) is the centroid of the image and is defined as:

$$x_0 = \frac{\sum_{i=1}^N \sum_{j=1}^N x_i f(x_i, y_j)}{\sum_{i=1}^N \sum_{j=1}^N f(x_i, y_j)}, \quad y_0 = \frac{\sum_{i=1}^N \sum_{j=1}^N y_j f(x_i, y_j)}{\sum_{i=1}^N \sum_{j=1}^N f(x_i, y_j)}. \quad (8)$$

To compute these central moments (Chong et al., 2004) expressed the translated Legendre polynomials in terms of the original Legendre polynomials according to the relation:

$$P_p(x-x_0) = \sum_{k=0}^p v_{p(p-k)} P_{p-k}(x). \quad (9.1)$$

$$P_q(y-y_0) = \sum_{m=0}^q \tau_{q(q-m)} P_{q-m}(y), \quad (9.2)$$

where the matrices $v_{p(p-n)}$ and $\tau_{q(q-m)}$ are centroid-dependent defined by using the following equations:

$$v_{pp} = 1, \quad (10.1)$$

$$v_{p(p-k)} = \frac{1}{B_{(p-k)(p-k)}} \times \left[\sum_{r=1}^k \binom{p-k+r}{r} (-x_0)^r B_{p(p-k+r)} - \sum_{s=1}^{k-1} v_{p(p-s)} B_{(p-s)(p-k)} \right], \quad (10.2)$$

$$\tau_{qq} = 1, \quad (10.3)$$

$$\tau_{q(q-m)} = \frac{1}{B_{(q-m)(q-m)}} \times \left[\sum_{d=1}^m \binom{q-m+d}{d} (-y_0)^d B_{q(q-m+d)} - \sum_{u=1}^{m-1} \tau_{q(q-u)} B_{(q-u)(q-m)} \right], \quad (10.4)$$

Subject to the conditions, $k-r$ even, $k-s$ even and $k \geq 1$; $m-d$ even, $m-u$ even and $m \geq 1$. Using Eq. (9) in Eq. (7) yields the following central 2D Legendre moments:

$$\varphi_{pq} = \sum_{k=0}^p \sum_{m=0}^q \frac{\lambda_{pq}}{\lambda_{(p-k)(q-m)}} v_{p(p-k)} \tau_{q(q-m)} \tilde{L}_{(p-k)(q-m)}. \quad (11)$$

In case of 3D objects, Eq. (11) is generalized to the following form:

$$\varphi_{pqr} = \sum_{k=0}^p \sum_{m=0}^q \sum_{n=0}^r \frac{\lambda_{pqr}}{\lambda_{(p-k)(q-m)(r-n)}} v_{p(p-k)} \tau_{q(q-m)} \eta_{r(r-n)} \tilde{L}_{(p-k)(q-m)(r-n)}. \quad (12)$$

where λ_{pqr} is a straightforward extension of that is defined by Eq. (6), and $\eta_{r(r-n)}$ is a centroid-dependent deduced by equations similar (10.1) and (10.2).

2.2. Scale invariants of Legendre moments

Non-uniform scale Legendre moments are defined as:

$$\psi_{pq} = \frac{(2p+1)(2q+1)}{4} \int_{-1}^1 \int_{-1}^1 P_p(ax) P_q(by) f(x, y) dx dy \quad (13)$$

where a and b are unequal non-zero real numbers representing the scaling factors in x - and y -direction respectively. Chong et al. (2004) expressed the scaled Legendre polynomials in terms of the original Legendre polynomials as follows:

$$\sum_{n=0}^p \delta_{pn} P_n(ax) = a^p \sum_{n=0}^p \delta_{pn} P_n(x), \quad (14.1)$$

$$\sum_{d=0}^q \delta_{qd} P_d(by) = b^q \sum_{d=0}^q \delta_{qd} P_d(y), \quad (14.2)$$

where the matrix $\delta_{p,n}$ is defined by using the following equations:

$$\delta_{pp} = 1, \quad (15.1)$$

$$\delta_{pn} = \sum_{r=0}^{p-n-2} -\frac{B_{(p-r)n} \delta_{p(p-r)}}{B_{nn}}. \quad (15.2)$$

Subject to the conditions, $p-n-r$ even, $p-n$ even and $p-n \geq 2$. Using Eq. (14) in Eq. (13) yields the following scaled Legendre moments:

$$\psi_{pq} = a^{p+1} b^{q+1} \sum_{n=0}^p \sum_{d=0}^q \frac{\lambda_{pq}}{\lambda_{nd}} \delta_{pn} \delta_{qd} \tilde{L}_{nd}. \quad (16)$$

The scaling factors a and b could be cancelled out. The normalized scale invariants of Legendre moments are subsequently derived as follows:

$$\omega_{pq} = \frac{\psi_{pq} \psi_{00}^{\zeta+1}}{\psi_{(p+\zeta)0} \psi_{0(q+\zeta)}}, \quad p, q \text{ and } \zeta = 0, 1, 2, 3, \dots \quad (17)$$

3. Refined translation-scale Legendre moment invariants

In this section, a refined version of the translation-scale 2D Legendre moments is presented. The exact central Legendre moments are simply computed by using a straightforward method which ignores the time-consuming calculation of the Eq. (10). In our recent work (Hosny, 2007b), a novel method was proposed for exact and fast computation of 2D Legendre moments. This method is adapted to compute the central 2D Legendre moments exactly without the need of original 2D Legendre moment computation.

A digital image of size $M \times N$ is an array of pixels. Centers of these pixels are the points (x_i, y_j) , where the image intensity function is defined only for this discrete set of points $(x_i, y_j) \in [-1, 1] \times [-1, 1]$. $\Delta x_i = x_{i+1} - x_i = 2/M$, $\Delta y_j = y_{j+1} - y_j = 2/N$ are sampling intervals in the x - and y -directions respectively.

The set of 2D central Legendre moment could be exactly computed by:

$$\hat{L}_{pq} = \sum_{i=1}^M \sum_{j=1}^N I_p(x_i - x_0) I_q(y_j - y_0) f(x_i, y_j), \quad (18)$$

Eq. (18) is valid only for $p \geq 1$, and $q \geq 1$. For $p = 0$ and $q = 0$ the following special cases are considered:

$$p = 0; \quad q = 0, 1, 2, 3, \dots, \text{Max};$$

$$\hat{L}_{0q} = \frac{1}{M} \sum_{i=1}^M \sum_{j=1}^N I_q(y_j - y_0) f(x_i, y_j), \quad (19)$$

$$q = 0; \quad p = 0, 1, 2, 3, \dots, \text{Max};$$

$$\hat{L}_{p0} = \frac{1}{N} \sum_{i=1}^M \sum_{j=1}^N I_p(x_i - x_0) f(x_i, y_j), \quad (20)$$

where

$$I_p(x_i - x_0) = \frac{(2p+1)}{(2p+2)} [(U_{i+1} - x_0)P_p(U_{i+1} - x_0) - P_{p-1}(U_{i+1} - x_0) - (U_i - x_0)P_p(U_i - x_0) + P_{p-1}(U_i - x_0)], \quad (21)$$

$$I_q(y_j - y_0) = \frac{(2q+1)}{(2q+2)} [(V_{j+1} - y_0)P_q(V_{j+1} - y_0) - P_{q-1}(V_{j+1} - y_0) - (V_j - y_0)P_q(V_j - y_0) + P_{q-1}(V_j - y_0)], \quad (22)$$

and the upper and lower limits are defined as:

$$U_{i+1} = x_i + \frac{\Delta x_i}{2} = -1 + i\Delta x, \quad (23.1)$$

$$U_i = x_i - \frac{\Delta x_i}{2} = -1 + (i-1)\Delta x, \quad (23.2)$$

$$V_{j+1} = y_j + \frac{\Delta y_j}{2} = -1 + j\Delta y, \quad (23.3)$$

$$V_j = y_j - \frac{\Delta y_j}{2} = -1 + (j-1)\Delta y, \quad (23.4)$$

The recurrence relation (4) is used to generate Legendre polynomial $P_p(x_i)$. In order to generate $P_p(U_{i+1} - x_0)$ and $P_q(V_{j+1} - y_0)$; $(U_{i+1} - x_0)$ and $(V_{j+1} - y_0)$ are used instead of x_i and y_j . Duplicated kernel generation time could be avoided where the polynomial $P_p(U_i - x_0)$ could be generated from $P_p(U_{i+1} - x_0)$ by using the following algorithm:

```

g3x (1,0) = 1.0;
g3x (1,1) = -1.0-x0;
for k = 1 to Max
    g3x (1,k+1) = (2 k + 1)/(k + 1)*(-1.0-x0)*g3x (1,k)-k/(k + 1)*
    g3x (1,k-1);
endfor
for i = 2 to N
    for k = 0 to Max
        g3x (i,k) = g2x (i-1,k)
    endfor
endfor
    
```

where $g2x$, $g3x$ are matrix representations of $P_p(U_{i+1} - x_0)$ and $P_p(U_i - x_0)$ respectively, N is the image size, and Max is the maximum moment order. The polynomial $P_q(V_j - y_0)$ will be generated from $P_q(V_{j+1} - y_0)$ using the same algorithm.

Computational times required to compute exact central Legendre moments by using Eq. (18) could be significantly reduced. The two-dimensional $(p+q)$ -order central Legendre moments can be obtained in two steps by successive computation of the 1D q th order moment for each row. This approach was successfully implemented in (Hosny, 2007b,c). A fast method for exact central Legendre moments computation will be presented. Eq. (18) will be rewritten in a separated form as follows:

$$\hat{L}_{pq} = \sum_{i=1}^M I_p(x_i - x_0) Y_{iq}, \quad (24.1)$$

where

$$Y_{iq} = \sum_{j=1}^N I_q(y_j - y_0) f(x_i, y_j). \quad (24.2)$$

Y_{iq} in Eq. (24.2) is the q th order moment of row i . Since,

$$I_0(x_i - x_0) = 1/M, \quad (24.3)$$

Substitutes Eq. (24.3) into (24.1), yields;

$$\hat{L}_{0q} = \frac{1}{M} \sum_{i=1}^M Y_{iq}. \quad (24.4)$$

Using the exact 2D central Legendre moments \hat{L}_{pq} defined by Eq. (24) in the Eqs. (16) and (17) yields the refined set of translation-scale 2D Legendre moments:

$$\hat{\psi}_{pq} = a^{p+1} b^{q+1} \sum_{n=0}^p \sum_{d=0}^q \delta_{pn} \delta_{pd} \hat{L}_{nd}, \quad (25.1)$$

$$\hat{\omega}_{pq} = \frac{\hat{\psi}_{pq} \hat{\psi}_{00}^3}{\hat{\psi}_{(p+2)0} \hat{\psi}_{0(q+2)}}. \quad (25.2)$$

4. Numerical experiments and computational complexity

In this section a verification of the proposed method is presented. The computational complexity of the proposed method for refined Legendre moment invariants and the conventional method of Chong are analyzed. To verify the proposed method, a gray scale image of baboon of size 256×256 is used. The original image in Fig. 1a is transformed with different scaling factors as in Fig. 1b and c. The second- and the third-order of the refined translation-scale 2D Legendre moment invariants defined by the Eq. (25) are computed and tabulated in Table 1.

The average elapsed CPU time in seconds is evaluated for the proposed refined method and the conventional one of Chong. The standard gray level images showed in Fig. 2a, b, c and d are used in this experiment. The original images are of size 512×512 . A set of contracted images are obtained by half size scaling. These images are of the size 256×256 , 128×128 , 64×64 and 32×32 . The average elapsed CPU for computing the first seven descriptors for the different versions of standard images are shown in Table 2. The last column of Table 2 represents the execution-time improvement ratio (ETIR) (Hosny, 2008). This criterion is expressed by the following equation:

$$ETIR = \left(1 - \frac{Time1}{Time2}\right) \times 100, \quad (26)$$

where $Time1$ and $Time2$ represent the elapsed CPU time required by the proposed and Chong's method respectively.

4.1. Computational complexity

Computational complexity is a crucial point in all aspects of image and pattern recognition. In this subsection, there is an analysis

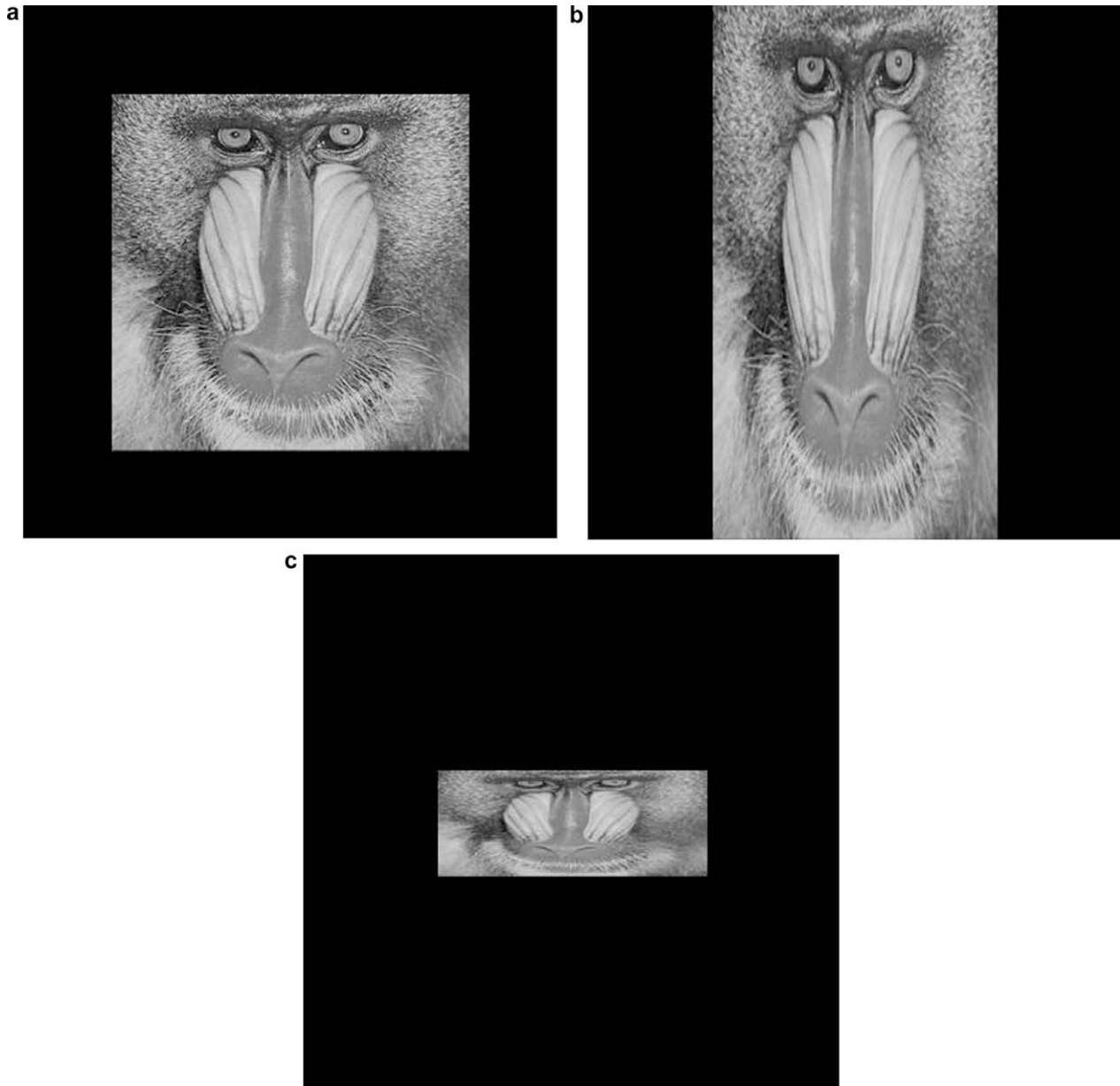


Fig. 1. (a) The original image (b) and (c) are transformed images.

Table 1
First and second order scale invariants for the baboon image.

| LMI's | Original image | First transformed image | Second transformed image |
|---------------------|----------------|-------------------------|--------------------------|
| $\hat{\omega}_{20}$ | 0.3554 | 0.3554 | 0.3554 |
| $\hat{\omega}_{11}$ | -7.9448 | -7.9448 | -7.9448 |
| $\hat{\omega}_{02}$ | 0.3029 | 0.3029 | 0.3029 |
| $\hat{\omega}_{30}$ | 0.0623 | 0.0623 | 0.0623 |
| $\hat{\omega}_{21}$ | 0.0894 | 0.0894 | 0.0894 |
| $\hat{\omega}_{12}$ | 0.3988 | 0.3988 | 0.3988 |
| $\hat{\omega}_{03}$ | 0.1963 | 0.1963 | 0.1963 |

of the computational complexity of both the refined and conventional methods.

The computation of translation-scale Legendre invariants by using the conventional method (Chong et al., 2004) consists of three main steps. These steps are namely, the approximate com-

putation of Legendre moments, the computation of translated Legendre moment invariants through the translated Legendre polynomials and the computation of the scaled Legendre moment invariants through the scaled Legendre polynomials.

In the first step, a set of approximate Legendre moments are computed and stored for the use in the next step. For a digital gray level image of size $N \times N$, and Max is moment order, the ZOA method for approximate Legendre moment's computation required $0.5(Max + 1)(Max + 2)N^2$ addition operations, and $(Max + 1)(Max + 2)N^2$ multiplication operations. These moments are image-dependent; therefore their values are computed for each image.

In the second step, central Legendre moments are calculated as a combination of translation matrices $v_{p(p-n)}$ and $\tau_{q, (q-m)}$; the normalization terms $\lambda_{p, q}$ and the approximated Legendre moments by using Eq. (11).

Each of the translated Legendre polynomials $P_p(x - x_0)$ and $P_q(y - y_0)$ is evaluated as a combination of the corresponding original polynomial as in the Eqs. (9). It is clear that, the computation of



Fig. 2. Standard image of “Lena” of size 512×512 .

Table 2

The execution-time improvement ratio.

| Image size | Chong's method | Proposed method | ETIR (%) |
|------------------|----------------|-----------------|----------|
| 512×512 | 5.1460 | 0.4060 | 92 |
| 256×256 | 0.8750 | 0.1560 | 82 |
| 128×128 | 0.2350 | 0.0620 | 74 |
| 64×64 | 0.1410 | 0.0470 | 67 |
| 32×32 | 0.093 | 0.0420 | 55 |

these polynomials is image-dependent. Consequently, their values are different from each image. Eqs. (9) required $(Max - 1)(Max + 2)$ additions and $Max(Max - 1)$ multiplications plus the computational process of original Legendre polynomials by using the recurrence relation.

According to Eq. (10), computational process of the matrices $v_{p(p-n)}$ and $\tau_{q(q-m)}$ is complicated and time-consuming. The main problem is that, these matrices are image-dependent, which means repetitive computation for each different image. Evaluation of these matrices required:

$$\frac{Max}{2}(Max - 1) + 2 \sum_{t=2}^{Max} \frac{t(t-1)}{2} + 2 \sum_{t=2}^{Max} \frac{(t-1)(t-2)}{2}, \quad \text{additions,} \quad (27.1)$$

$$\frac{Max}{2}(Max + 1) + 4 \sum_{t=2}^{Max} \frac{t(t+1)}{2} + 2 \sum_{t=2}^{Max} \frac{t(t-1)}{2}, \quad \text{multiplications,} \quad (27.2)$$

$$2 \sum_{t=2}^{Max} \frac{t(t+1)}{2}, \quad \text{power functions.} \quad (27.3)$$

In addition to these operations, computation of the Legendre coefficient matrix $B_{k,p}$ defined by Eq. (5) is required. Computation of factorial terms is essential.

The third step is concerned with the computation of the scaled Legendre moment invariants. In this step, the scaled Legendre polynomials are related to the original polynomials through the scaling matrix $\delta_{p,n}$. Fortunately, this matrix is image-independent

Table 3
Complexity analysis of the two methods For digital image of size $N \times N$ and a maximum moment order equal to Max .

| | Chong's method, 2004 | Refined method |
|-----------------|---|---|
| Multiplications | $(Max + 1)(Max + 2)N^2 + Max(Max - 1) + \frac{1}{2}Max(Max + 1) + 4\sum_{t=2}^{Max} \frac{t(t+1)}{2} + 2\sum_{t=2}^{Max} \frac{t(t-1)}{2}$ | $\frac{1}{2}NMax(2N + Max - 1) + N^2$ |
| Additions | $\frac{1}{2}(Max + 1)(Max + 2)N^2 + (Max - 1)(Max + 2) + \frac{1}{2}Max(Max - 1) + 2\sum_{t=2}^{Max} \frac{t(t-1)}{2} + 2\sum_{t=2}^{Max} \frac{(t-1)(t-2)}{2}$ | $\frac{1}{2}(N - 1)(Max + 1)(2N + Max + 2)$ |
| Power functions | $2\sum_{t=2}^{Max} \frac{t(t+1)}{2}$ | – |

Table 4
Number of addition and multiplication operations required by both methods.

| Image size | Max | Chong's method | Proposed method |
|-------------|-----|-----------------|-----------------|
| 1024 × 1024 | 10 | 69,206,749 (+) | 11,590,590 (+) |
| | | 138,413,491 (*) | 11,580,416 (*) |
| 1024 × 1024 | 6 | 29,360,299 (+) | 7,361,508 (+) |
| | | 58,720,637 (*) | 7,355,392 (*) |
| 512 × 512 | 10 | 17,302,237 (+) | 2,911,678 (+) |
| | | 34,604,467 (*) | 2,906,624 (*) |
| 512 × 512 | 6 | 7,340,203 (+) | 1,845,732 (+) |
| | | 14,680,445 (*) | 1,842,688 (*) |
| 256 × 256 | 10 | 4,326,109 (+) | 734,910 (+) |
| | | 8,652,211 (*) | 732,416 (*) |
| 256 × 256 | 6 | 1,835,179 (+) | 464,100 (+) |
| | | 3,670,397 (*) | 462,592 (*) |

which means that, it could be pre-computed, stored and recalled whenever it is needed to avoid any repetitive computation. Therefore, the computational complexity of this step is excluded from the comparison.

The scaled Legendre moments and the aspect ratio invariants are computed according to Eqs. (16) and (17) as a combination of the scaling matrix $\delta_{p,n}$ and the matrix $\delta_{q,a}$; the normalization terms $\lambda_{p,q}$ and the approximated Legendre moments. It is easy to note that, the computation process of the translation invariants is the most time-consuming part of the whole computational process.

On the other side, the proposed method reduces the computational complexity tremendously through the direct computation of the exact central Legendre moments by using an adapted method from the original exact method (Hosny, 2007b). According to Eq. (18), the computational complexity of the exact central Legendre moment's computation is similar to that one for the exact Legendre moments. Based on the detailed discussion of the computational complexity in (Hosny, 2007b), the computational process of the exact central Legendre moments required $0.5(Max + 1)(N - 1)(2N + Max + 2)$ addition operations and $0.5N Max(2N + Max - 1) + N^2$ multiplication operations. These operations are much less than that of the required operations to compute only the approximate ZOA Legendre moments. The total addition and multiplications process required by the method of Chong and the proposed refined method to compute central Legendre moments are summarized in Table 3.

For more clarity, Table 4 shows the total number of only addition and multiplication operations for few standard images of different sizes and different orders. A quick comparison ensures the huge difference between the computational complexities of these two methods. It must be noted that, in the refined method, there

is no need to compute the normalization terms $\lambda_{p,q}$, the coefficient matrix $B_{k,p}$, the power functions nor the factorial terms. This ensures the efficiency of the proposed method.

5. Conclusion

This work proposes a refined method to compute translation-scale Legendre moment invariants for gray-scale images. Direct computation of the exact central Legendre moments avoids the complicated computation of the translation matrices. The numerical experiments show the validity of the computed values, while the analysis of the computational complexity ensures the significance of the proposed refined method.

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