Accurate Computation of ART Coefficients for Binary Images

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Abstract—**The coefficients of the Angular Radial Transform (ART) are used as MPEG-7 Shape descriptors where the bases functions ART are defined in polar coordinates over a unit disk. Conventional computational methods for computing the ART coefficients are depending on a circular to square mapping which produce two kinds of errors and results in inaccurate set descriptors. This work proposes accurate and efficient method for computing ART coefficients in polar coordinates. The unit disk is divided into a number of non-overlapped circular rings that are divided into circular sectors of the same area. Each sector is represented by one point in its centre. Computation process in the polar domain completely removes both approximation and geometrical errors produced by the conventional methods. Symmetry properties are applied to significantly reduce the computational complexity. Numerical experiments are performed to ensure the efficiency of the proposed method.**

Keywords: Angular radial transform; polar coordinates; Accurate MPEG-7 Shape descriptors; Binary images; fast computation

I. INTRODUCTION

The number of generated digital images is extremely increased. In order to find an image, the image has to be described or represented by certain features. Shape is an important visual feature of an image/object and it is one of the basic features used to describe image content. Searching for images using shape features has attracted much attention, where shape representation and description is an attractive task. Trademark retrieval system is a typical example for shape representation and description [1]. Zhang and Lu presented an excellent review of shape representation and description techniques [2].

Angular Radial Transform (ART) is a region-based shape descriptor for MPEG-7 [3]. ART bases functions are orthogonal and defined in polar coordinates in a separable form along both radial and angular directions [4]. Therefore, it can represent the shape of object in an image with minimum amount of information redundancy. Away from this attractive property, ART has additional attractive characteristics, where it could represent shape information using a small number of features, robust against noise and is rotationally invariant [5-6].

In order to reduce the computational complexity, two algorithms have been proposed. The first one, Hwang and Kim [7] applied the idea of symmetry and anti-symmetry in order to reduce the huge arithmetic demands of the sinusoidal functions. This approach reduces the computational complexity to 25%. Kotoulas and Andreads [8] proposed the use of Chebyshev polynomials of first and second kinds to represents the angular part of the ART basis functions. Unfortunately, both methods compute approximate ART coefficients and consequently, produced inaccurate coefficient sets. Therefore, we can conclude that, these methods achieve one goal and failed in the other.

This paper proposes accurate, fast and memory-efficient method for computing ART coefficients. The coefficient set of 2D ART are computed accurately in polar coordinates. The symmetry property is applied where more than 87% of computational complexity is reduced. According to the separable property of the ART basis function, a fast algorithm is applied to accelerate the computation of ART coefficients.

The rest of the paper is organized as follows: In section II, an overview of ART and the conventional approximate method. Accurate computation of ART coefficients in Polar Coordinates is described in section III. Section IV is devoted to numerical experiments. Conclusion and concluding remarks are presented in section V.

II. ANGULAR RADIAL TRANSFORM

A. Coefficients of ART

The ART coefficients of order p and q for an image intensity function $f(x,y)$ are defined by projecting the input image onto the ART basis functions and defined as follows:

$$F_{pq} = \int_0^{2\pi} \int_0^1 V^*{}_{pq}(r,\theta) f(r,\theta)\ r\,dr\,d\theta \qquad (1)$$

The asterisk symbol refers to the complex conjugate. The ART basis functions $V_{pq}(r,\theta)$ of order p and q are continuous orthogonal polynomials defined in polar coordinates over a unit disk. These polynomials are expressed in a separable form of both radial and angular parts as follows:

$$V_{pq}(r,\theta) = R_p(r)\,A_q(\theta) \qquad (2)$$

The indices p and q are nonnegative integers. The real-valued radial polynomial $R_p(r)$ and the angular basis function are defines as:

$$R_p(r) = \begin{cases} 1, & p = 0 \\ 2\cos(p\pi r), & \textit{otherwise} \end{cases} \qquad (3)$$

$$A_q(\theta) = \frac{1}{2\pi}\exp(\hat{i}q\theta) \qquad (4)$$

With $\hat{i} = \sqrt{-1}$. One of the important characteristics of ART is the rotational invariance. The magnitude values of ART coefficients are unaffected and remain identical for image functions before and after rotation. The original image is represented by the intensity image function $f(r,\theta)$ .A counterclockwise rotation by angle α results in the transformed image intensity function $g(r,\theta) = f(r,\theta-\alpha)$. The ART coefficients of original and rotated images are F_{pq} and $F^{rot}{}_{pq} = \exp(-\hat{i}q\alpha)F_{pq}$. The magnitude values are the same, where $\left\|\exp(-\hat{i}q\alpha)\right\| = 1$. Based on this characteristic, ART are used as shape descriptors in MPEG-7.

B. Approximate computation of ART coefficients

Approximate computation of ART coefficients generally based on circle-to-square mapping where the ART basis functions are deified in polar coordinates over a unit disk and digital images are usually defined in Cartesian coordinates. Such mapping produces two sources of errors [9], namely geometrical and numerical errors. The propagation of these errors in numerical computations degrades the accuracy of the computed coefficients. The ART coefficients of order p and q for an image intensity function $f(x,y)$ are approximately computed using the zeroth order approximation (ZOA), where, the integrals in equation (1) are replaced by summations and the image is normalized inside the unit disk. The approximate ART coefficients are:

$$\tilde{F}_{pq} = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} R_{pq}(r_{xy})\exp(-\hat{i}q\theta_{xy}) f(x,y) \qquad (5)$$

With $r_{xy} = \sqrt{x^2 + y^2}$ and $\theta_{xy} = \arctan(y/x)$.

III. ACCURATE COMPUTATION OF ART COEFFICIENTS

A. Image mapping into polar coordinates

Computation of ART coefficient in polar coordinates based on the idea of splitting the unit disk into a set of non-overlapped circular sectors. For input image of size $N \times N$ as depicted in figure (1), a new image mapping is proposed where all of the required computational processes are done in the polar coordinates. This mapping method is a modification of the method described in [10]. In this new mapping method, the total number of input Cartesian image pixels is equal to the number of mapped circular sector pixels as depicted in figure (2).

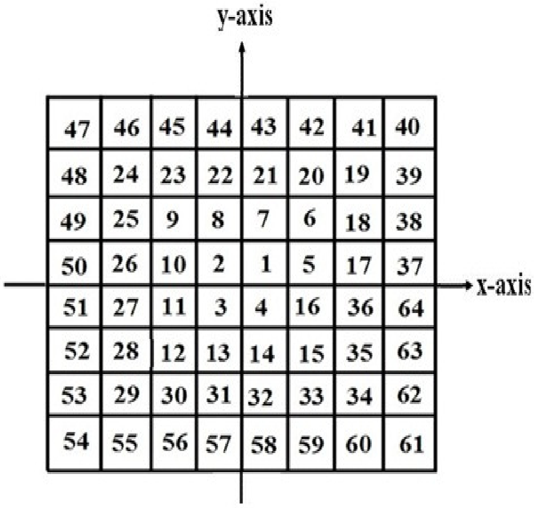

Figure 1. Cartesian image pixels

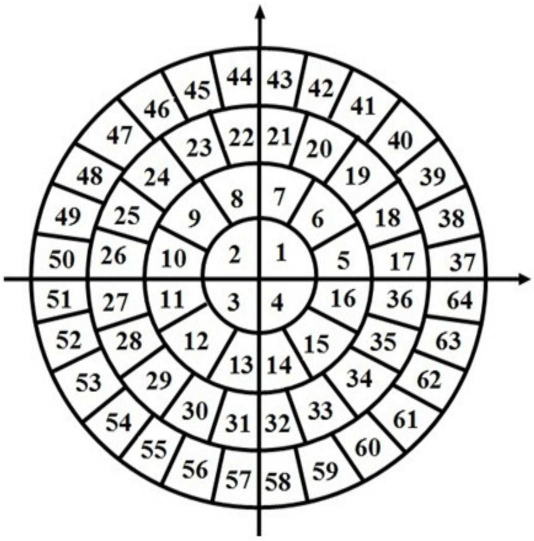

Figure 2. Mapped polar image pixels

The new mapping method is very simple and could be achieved by applying the following steps:

1- The unit disk is divided into, $N/2$, concentric, non-overlapped circular rings.

2- Each circular ring is divided into, $(8i+4)$, circular sectors where $i = 0,1,2,......,N/2-1$ and $i = 0$ refers to the innermost circular ring.

3- The different values of the angle θ could be created by using the following pseudo-code:

for $i = 0$ **to** $N/2-1$
  for $j = 0$ **to** $(8i+4)-1$
    $\theta_{i,j} = 2\pi(j+0.5)/(8i+4)$
  endfor
endfor

B. Fast computation of accurate ART coefficients

The 2D ART coefficients of order p and q are defined as:

$$F_{pq} = \frac{1}{2\pi} \sum_i \sum_j \hat{f}\left(r_i, \theta_{i,j}\right) H_{pq}\left(r_i, \theta_{i,j}\right) \tag{6}$$

With:

$$H_{pq}\left(r_i, \theta_{i,j}\right) = I_p\left(r_i\right) I_q\left(\theta_{ij}\right) \tag{7}$$

$$I_p\left(r_i\right) = \int_{U_i}^{U_{i+1}} R_p(r) r\, d r \tag{8}$$

$$I_q\left(\theta_{ij}\right) = \int_{V_{i,j}}^{V_{i,j+1}} \exp\left(-\hat{i} q \theta\right) d\theta \tag{9}$$

The image intensity function $\hat{f}\left(r_i, \theta_{i,j}\right)$ is deduced from the input image intensity function using the cubic interpolation [11]. The upper and lower limits are defined by using the following vectors:

$$\begin{aligned} U_{i+1} &= (2i+1)/N\ ; \quad V_{i,j+1} = \theta_{i,j} + \pi/(8i+4) \\ U_i &= 2i/N \quad\quad\ ; \quad V_{i,j} = \theta_{i,j} - \pi/(8i+4) \end{aligned} \tag{10}$$

Using equation (3) into (8), we can write the real-valued radial-based integral as:

$$I_p\left(r_i\right) = \int_{U_i}^{U_{i+1}} 2\cos\left(p\pi r\right) r\, d r \tag{11}$$

Equation (11) is applicable only for $p \neq 0$. The special case of $p = 0$ is represented as follows:

$$I_0\left(r_i\right) = \left(U^2{}_{i+1} - U^2{}_i\right)/2 \tag{12}$$

Applying the rule of integration by parts [12], we can analytically evaluate the integral $I_p(r_i)$ as follows:

$$I_p(r_i)=\left(\frac{2r}{p\pi}\sin(p\pi r)+\frac{2}{(p\pi)^2}\cos(p\pi r)\right)_{U_i}^{U_{i+1}} \quad (13)$$

The angle-based integral in equation (11) can be rewritten as:

$$I_q(\theta_{i,j})=\frac{\hat{i}}{q}\left(\exp\left(-\hat{i}qV_{i,j+1}\right)-\exp\left(-\hat{i}qV_{i,j}\right)\right) \quad (14)$$

It is clear that, the kernels $I_p(r_i)$ and $I_q(\theta_{i,j})$ are independent from the image. Both kernels can be pre-computed and stored for any future use.

Computational complexity of equation (6) could be significantly reduced. Two types of symmetry are applied, where only one-eighth of the whole circular domain is used in the computational process. The symmetry property was discussed in details in [10]. Fast computation could be achieved by successive computation in angular and radial directions. Equation (6) will be rewritten in a separable form as follows:

$$F_{pq}=\frac{1}{2\pi}\sum_i I_p(r_i)Y_{iq} \quad (15)$$

With:

$$Y_{iq}=\sum_j I_q(\theta_{ij})f(r_i,\theta_{ij}) \quad (16)$$

IV. NUMERICAL EXPERIMENTS

The validity of the proposed method could be proved by using a test image. The considered image is represented by a unit circle with a white upper half and a black lower half as depicted in figure (3). The intensity function of this image is defined as:

$$f(r,\theta)=\begin{cases}1, & 0\le\theta\le\pi\\ 0, & \pi<\theta\le 2\pi\end{cases} \quad (17)$$

ART coefficients for the considered image are defined as:

$$F_{pq}=\frac{1}{\pi}\int_0^{\pi}\int_0^{1}\cos(p\pi r)\exp\left(-\hat{i}q\theta\right)r\,dr\,d\theta \quad (18)$$

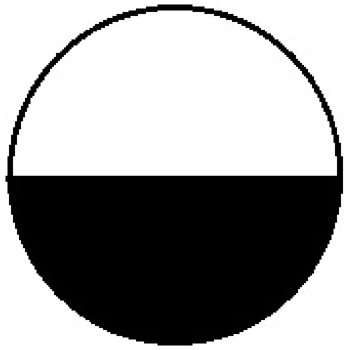

Figure 3: Binary test image

These ART coefficients could be exactly computed by using the principles of integral calculus. Equation (18) is rewritten as follows:

$$F_{pq}=\frac{1}{\pi}\left(\int_0^{\pi}\exp\left(-\hat{i}q\theta\right)d\theta\right)\left(\int_0^{1}\cos(p\pi r)\,r\,dr\right) \quad (19)$$

The evaluation of the first integral is straightforward. The rule of integration by parts is used in the evaluation of the second integral. Since, $\sin(p\pi)=0$ for integer values of p, the integration of equation (19) yields:

$$F_{pq}=\frac{\hat{i}}{q\pi}\left(\exp\left(-\hat{i}q\pi\right)-1\right)\left((\cos(p\pi)-1)/p^2\pi^2\right) \quad (20)$$

Based on the principles of trigonometric [12], $\cos(p\pi)=(-1)^p$; therefore, the value of F_{pq} defined by equation (20) depends on whatever the value of p is even or odd and rewritten as follows:

$$F_{pq}=\begin{cases}0, & p\ is\ even\\ \dfrac{-2\hat{i}}{q\,p^2\pi^3}\left(\exp\left(-\hat{i}q\pi\right)-1\right), & p\ is\ odd\end{cases} \quad (21)$$

Since, the ART coefficients are rotation invariants, the scaling and translation invariance could be achieved easily. The center of mass of the input image is moved to coincide with the center of the unit disk. Then, the input image is bounded by the unit circle. A 36 ART coefficients of order $0\le p<3$ and $0\le q<12$ were computed [7]. According to equation (21), the values of 24 coefficients are equal zero while only 12 coefficients are non-zero. The magnitude values of the non-zero ART coefficients are listed in table (1).

TABLE 1. MAGNITUDE VALUES OF THE ART COEFFICIENTS

Theoretical	Computational
0.0617	0.0617
0.0322	0.0322
0.0215	0.0215
0.0161	0.0161
0.0129	0.0129
0.0108	0.0108
0.0092	0.0092
0.0081	0.0081
0.0072	0.0072
0.0065	0.0065
0.0059	0.0059
0.0054	0.0054

V. CONCLUSION

This paper proposes a fast, accurate and memory-efficient method for computing 2D ART coefficients in polar coordinates. The unit disk is divided into concentric non-overlapped circular rings. All these rings are divided with a systematic process to produce a set of equally-area circular sectors, where each of these sectors is represented by only one point in its centre. Symmetry property is applied in order to reduce the computational complexity by approximately 87%. A fast algorithm is applied to significantly reduce the computational time. In general, the proposed method is fast, accurate and memory-efficient.

REFERENCES

[1] Y. S. Kim and W. Y. Kim, “Content-based trademark retrieval system using visually salient feature,” Image and Vision Computing, vol. 16, pp. 931–939, Aug. 1998.

[2] D. Zhang and G. Lu, "Review of shape representation and description techniques," Pattern Recognition, vol. 37, pp.1–19, Jan. 2004.

[3] M. Bober, “MPEG-7 visual shape descriptors,” IEEE Trans. Circuits Syst. Video Technol., vol. 11, pp. 716–719, June. 2001.

[4] W. Y. Kim and Y. S. Kim, “A new region-based shape descriptor: The ART (angular radial transform) descriptor,” presented at the ISO/IEC, Maui, HI, Dec. 1999.

[5] J. Fang, and G. Qiu, "Human face detection using angular radial transform and support vector machines," In proceeding of the international conference on image processing, Vol. I, pp. 669–672, 2003.

[6] M.-S. Choi and W. -Y. Kim, "The description and retrieval of a sequence of moving objects using a shape variation map," Pattern Recognition Letters, Vol. 25, pp.1369–1375, Sep. 2004.

[7] S. Hwang and W. Kim, "Fast and Efficient Method for Computing ART," IEEE Transaction on Image processing, vol. 15, pp. 112-117, Jan. 2006.

[8] L. Kotoulas and I. Andreadis “An Efficient Technique for the Computation of ART,” IEEE Trans. Circuits Syst. Video Technol., Vol. 18, pp. 682-686, May 2008.

[9] K. M. Hosny, "Fast Computation of Accurate Zernike moments," Journal of Real-Time Image Processing, vol. 3, No.2, p. 97-107, 2008.

[10] K. M. Hosny, M. A. Shouman, and H. M. Abdel Salam, "Fast computation of orthogonal Fourier–Mellin moments in polar coordinates," Journal of Real-Time Image Processing, Vol. 6, No.2, p. 73-80, 2011.

[11] Y. Xin, M. Pawlak, and S. Liao, "Accurate Computation of Zernike Moments in Polar Coordinates," IEEE Trans. Image Processing, vol. 16, pp. 581-587, 2007.

[12] J. W. Harris and H. Stocker, "Handbook of Mathematics and Computational Science", Springer-Verlag, New-York, 1998.